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Schauder fixed point theorem based existence of periodic solution for the response of Duffing's oscillator †

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Abstract

An initial-boundary value problem that is Duffing's oscillator with time varying coefficients will be studied. Using Banach's fixed-point theorem, the existence of periodic solution of the equation will be predicted. The method applied in this paper is the Schauder second fixed point theorem, which includes the response of structures under vibratory force systems. As an example, the dynamics of nonlinear simply supported rectangular thin plate under influence of a relatively moving mass is studied. By expansion of the solution as a series of mode functions, the governing equations of motion are reduced to an ordinary differential equation for time development vibration amplitude, which is Duffing's oscillator. Finally, a parametric study is developed, after that some numerical examples are solved, and the validity of the present analysis is clearly shown.

Keywords: Banach's theorem; Large deformation of thin plates; Moving loads; Non-linear vibration; Schauder fixed point theorem

1. Introduction

In recent years, there has been a marked interest in the solution-existence prediction methods. Fixed point theorems have been applied to diverse areas, so they have lasting power and been important theorems in nonlinear functional analysis. Especially, Brouwer, Kakutani and other fixed point theorems have played important roles in optimization theory, game theory, minimax problems, variational inequality problems, etc.

A large number of studies have been done in this area. Graef and Kong studied the nonlinear boundary value problem and obtained a necessary and sufficient condition for the existence of symmetric positive solutions [1]. Habib et al. presented existence results for the polyharmonic nonlinear elliptic boundary-value problem [2]. Some classical tools have been used to study singular equations in the literature. Some of these classical tools include some fixed point theorems in cones for completely continuous operators. From those, Torres studied differential equations via a Krasnoselskii fixed point theorem [3]. Latracha et al. analyzed the existence of solutions to a kind of boundary value problem for isotropic scattering kernels on Lp spaces for p=1 [4]. Benedikt proved the existence and uniqueness of a solution of the initial value problem for a special equation [5]. Various methods for studying the damped Duffing equation and the forced Duffing equation in stability [6-8], periodic solutions [9-12] and numerical simulations [13], etc., have been proposed and a vast number of profound results have been established. The global existence and the asymptotic behavior of solutions were undertaken by Kosecki et al. [14-16].

The Schauder fixed point theorem is, undoubtedly,

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one of the most important theorems of nonlinear analysis [17]. In this regard, the uniqueness of a time-periodic solution was proved by Gao and Guo using the Galerkin method as well as Leray-Schauder fixed point theorem [18]. Bai and Fang established the existence of a positive solution to a singular coupled system of nonlinear fractional differential equations [19]. Their analysis relies on a nonlinear alternative of Leray–Schauder type and Krasnoselskii's fixed point theorem in a cone. Chu and Torres [20] studied the existence of positive periodic solutions to second order singular differential equations. The proof relies on the Schauder fixed point theorem. Their results prove that in some situations weak singularities can help create periodic solutions, just as pointed out in [21]. By using the Schauder fixed point theorem and analysis method, Guo et al. established the existence of solutions for the m-point boundary value problem [22].

The possibility of obtaining the existence of solutions to various problems of physics and engineering, without the need for problem solving is quite appealing, in particular due to the reduction in time consumption and the time taken in preparing the data or analyzing the results.

The present paper deals with the determination of sufficient conditions for the existence of the periodic solution of the initial boundary value problems, which correspond, respectively, to the response of some mechanical systems/structures, via the application of the Schauder theorem. The periodicity of the overall nonlinear relations as Duffing's oscillator with time varying coefficients is investigated. While existence of the response is guaranteed in the resulting domain, the method does not say anything about the response outside this sufficient condition. This is illustrated by an example. Based on classical plate theory a rectangular plate on which a moving mass is traveling with specific frequency is considered. By expansion of the solution as a series of mode functions, the governing partial differential equation was converted into a nonlinear ordinary differential equation with time varying coefficients. The conditions guarantee the existence of a periodic response with the same frequency of the base excitation. The range of the parameters of the problem for which the existence of periodic solution is guaranteed is presented and the effects of the geometrical properties of the plate as well as the maximum allowable vibrating force to the minimum frequency of moving load are studied in detail.

2. Periodicity in solution

"Duffing Equations," Eq. (1), are a common form of equations with time dependent coefficients. In this case, it is considered that A as the response of a vibratory system is a function of two spatial variable as well as time.

$$\ddot{A} + P(t)A + Q(t)A^3 = F(t) \tag{1}$$

It is natural to ask whether Eq. (1) has a periodic solution for all periodic force. To answer this question, we assume that $P(t)$, $Q(t)$ and $F(t)$ are simultaneously odd (even) and periodic in t with period $T = 2\pi/\omega$.

Theorem: if the function F is continuous, odd, and periodic, then Eq. (1) has periodic solution of the same period as F.

Proof: We first simplify the problem by the rescaling so that the force has period 2. Define $s = \omega t/\pi$, then Eq. (1) becomes

$$-A'' = p(s)A + q(s)A^3 - f(s) \tag{2}$$

where $p = \beta^2.P$, $q = \beta^2.Q$, $f = \beta^2.F$ and $\beta = \pi/\omega$. The period of p, q and f is 2; we shall prove that there is an odd (even) periodic solution $A(s)$ such that $A(0) = 0$ and $A(-1) = A(1)$ (respectively for even functions).

The problem of finding a periodic solution can then be reduced to a two-point boundary-value problem on [0, 1]. Suppose that we have found a solution $A(s)$ of Equation (2) on [0, 1] satisfying $A(0) = A(1) = 0$. We can then extend it to [-1, 1] by defining $A(-s) = -A(s)$ for $0 \le s \le 1$, and then extend it to all s by making it periodic with period 2. It then satisfies Eq. (2) everywhere, and is the required periodic solution.

Thus, our problem is to solve Eq. (2) on the interval [0, 1] with homogeneous two-point boundary conditions.

We use Green's function to turn it into the integral equation

$$A(s) = \int_0^1 g(s,\zeta) f^*(\zeta, A(\zeta)) d\zeta \tag{3}$$

where:

$$f^*(\zeta, A(\zeta)) = -f(\zeta) + q(\zeta)A^3(\zeta) + p(\zeta)A(\zeta) \tag{4}$$

And g is Green's function for the operator $-D^2$ with boundary conditions $A(0)=A(1)=0$. Therefore:

$$g(s,\zeta)=\begin{cases} s(1-\zeta) & for \quad s\le\zeta \\ \zeta(1-s) & for \quad \zeta\le s \end{cases} \tag{5}$$

We work in the Banach space C [0,1]; let us define the operator $L:C[0,1]\to C[0,1]$ by

$$L(A)=\int_0^1 \beta^2 g(s,\zeta)(PA+QA^3-F)\,d\zeta \tag{6}$$

and use Schauder's second theorem. We are going to find a sphere as a closed convex set and choose its radius so that it is mapped into a relatively compact subset of itself by the integral operator of the equation

$$A(s)=\beta^2\int_0^1 g(s,\zeta).(PA+QA^3-F)\,d\zeta \tag{7}$$

We know that $f^*\left(=pA+qA^3-f\right)$ is not bounded, whereby both K and f are continuous. Let $\bar{M}$ be any positive number. Set:

$$S=\left\{A\in C[0,1]:\|A\|\le\bar{M}\right\} \tag{8}$$

Does L map S $\to$ S ?

Because of boundedness of the continuous real valued function $K(s,\zeta)=\beta^2 g(s,\zeta)$ on the compact set in a normed space [23], we have:

$$|K(s,\zeta)|=\beta^2|g(s,\zeta)|\le\beta^2.C \tag{9}$$

The maximum value of g is 1/4 so that:

$$|K(s,\zeta)|=\frac{\beta^2}{4}=\frac{\pi^2}{4\omega^2}=C^* \tag{10}$$

and we suppose that

$$\begin{aligned} B^*=\sup&\left\{\int\left|PA+QA^3-F\right|d\zeta:\|A\|\le\bar{M}\right\} \\ &\le P^*\bar{M}+Q^*\bar{M}^3+F^* \end{aligned} \tag{11}$$

in which:

$$\begin{aligned} &P^*=\max\{P(s):s\in[0,1]\} \\ &\text{and}\quad Q^*=\max\{P(s):s\in[0,1]\} \\ &\text{and}\quad F^*=\max\{F(s):s\in[0,1]\} \end{aligned} \tag{12}$$

S is not compact, so we cannot apply Schauder's first theorem; we must show that the set

$$R=\{LA:A\in S\} \tag{13}$$

is relatively compact and use Schauder's second theorem.

R is uniformly bounded because $\|LA\|\le B^*C^*$ for any A.

K is continuous on the closed square $0\le s,\zeta\le 1$ by the uniform continuity theorem [23]. Therefore:

$$\begin{aligned} &\forall\varepsilon>0 \quad \exists\delta>0\Rightarrow|K(s_1,\zeta)-K(s_2,\zeta)|<\frac{\varepsilon}{B^*} \\ &if \qquad |s_1-s_2|<\delta \end{aligned} \tag{14}$$

for any A:

$$\begin{aligned} &|(LA)(s_1)-(LA)(s_2)|= \\ &\left|\int_0^1[K(s_1,\zeta)-K(s_2,\zeta)]f^*(C,A(\zeta))d\zeta\right|< \\ &B^*\times\frac{\varepsilon}{B^*}=\varepsilon \end{aligned} \tag{15}$$

Thus R is equicontinuous and hence relatively compact by Arzela-Ascoli's theorem.

We show that L is a continuous operator. For each s:

$$\begin{aligned} &\|LU-LV\|= \\ &\left|\int_0^1 K(s,\zeta)[A(\zeta,U(\zeta))-A(\zeta,V(\zeta))]\,d\zeta\right|\le \\ &\int_0^1|K(s,\zeta)||A(\zeta,U(\zeta))-A(\zeta,V(\zeta))|\,d\zeta \end{aligned} \tag{16}$$

$A(\zeta,z)$ is continuous on the compact set $\left\{(\zeta,z)\in R^2\mid 0\le\zeta\le1,|z|\le2\|V\|\right\}$ and therefore uniformly continuous there by the uniform continuity theorem. Hence:

$$\forall\varepsilon>0 \quad \exists\delta>0 \quad ||U(\zeta)-V(\zeta)|<\delta \tag{17}$$

$$\begin{aligned} &\Rightarrow|A(\zeta,U(\zeta))-A(\zeta,V(\zeta))|<\varepsilon.C^* \\ &when \qquad |U(\zeta)-V(\zeta)|<\delta \end{aligned} \tag{18}$$

$$\begin{aligned} &\|LU-LV\|\le\varepsilon\int_0^1|K(s,\zeta)|d\zeta\times\frac{1}{C^*}\le \\ &\frac{\varepsilon}{C^*}\int_0^1 C^*dt=\varepsilon \end{aligned} \tag{19}$$

and L is continuous operator. Now, using Schauder's second fixed point theorem, the operator L has a fixed point.

Then, Banach's fixed point theorem can be used to prove the uniqueness of the periodic solutions. Let S

be a Banach space defined in Eq. (8). Rewriting Eq. (6) we have:

$$LA = \beta^2 \times \int_0^1 g(s,\zeta)\left\{-F(\zeta) + Q(\zeta)A^3 + P(\zeta)A\right\} d\zeta \tag{20}$$

The solution to be found is the fixed point of the above transformation.

Well

$$\left|(LA)(x)\right| \le C^* B^* \tag{21}$$

so,

$$\|LA\| \le B^* C^* \\ \Rightarrow LA \in S \quad if \quad B^* C^* \le \bar{M} \tag{22}$$

Thus L maps the whole space into the S set and $L: S \to S$, So that:

$$\bar{M} \ge \frac{\pi^2}{4\omega^2}\left(P^*\bar{M} + Q^*\bar{M}^3 + F^*\right) \ge B^* C^* \tag{23}$$

$$\Rightarrow \quad 4\omega^2\bar{M} - \pi^2 P^*\bar{M} - \pi^2 Q^*\bar{M}^3 \ge \pi^2 F^* \tag{24}$$

$$\left(\left(\frac{2\omega}{\pi}\right)^2 - P^*\right)\bar{M} - Q^*\bar{M}^3 \ge F^* \tag{25}$$

If $P^* \ge 4\omega^2/\pi^2$ or $\omega \le (\pi/2)\sqrt{P^*}$, the left-hand-side of the above equation is always negative, so the above inequality is never satisfied and no conclusion can be drawn. (There might be a solution even though we can't prove it by this method.)

If $\omega > (\pi/2)\sqrt{P^*}$, it may or may not possible to find M satisfying this inequality. Maximum value of left-hand-side of Eq. (25), as M is varied, is

$$2\left(4\omega^2 - \pi^2 P^*\right)^{\frac{3}{2}} \Big/ \left(3\pi^3\sqrt{3Q^*}\right) \ge F^* \tag{26}$$

$$\bar{M} = \sqrt{\frac{\left(4\omega^2/\pi^2\right) - P^*}{3Q^*}} \tag{27}$$

Hence, there is a solution if

$$2\left(4\omega^2 - \pi^2 P^*\right)^{\frac{3}{2}} \Big/ \left(3\pi^3\sqrt{3Q^*}\right) \ge F^* \tag{28}$$

i.e.,

$$F^* \le F_0^* = \frac{2\left(4\omega^2 - \pi^2 P^*\right)^{3/2}}{3\pi^3\sqrt{3Q^*}} \tag{29}$$

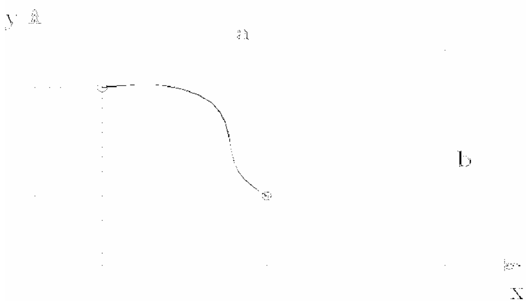

Fig. 1. Path definition of the moving mass traveling.

Thus if the forcing frequency $\omega > (\pi/2)\sqrt{P^*}$ (where the natural linear resonance frequency is 1) then there is a periodic solution with frequency ω if the amplitude of forcing term F^* is $\le F_0^*$: a limiting value which goes toward 0, as $\omega \to (\pi/2)\sqrt{P^*}$ and goes toward ∞, as $Q^* \to 0$. According to above discussion and substituting $\omega = 2\pi/T$ in relations, it can be inferred that there is a guarantee for the existence of periodic solution when $0 < T^2 P^* < 16$.

The fixed point theorem also guarantees the uniqueness of the solution, but it does not mean that under the conditions that the theorem holds, the system will not have any bifurcations. Furthermore, it guarantees the convergence of a successive approximation scheme constructed on Eq. (20), i.e., one can start with an arbitrary $A_0(t)$, put it in the right hand side of Eq. (20) to obtain $A_1(t)$ and go on iteratively and be sure that the series obtained is convergent.

3. Numerical results and discussion

In this section, an analysis is carried out in detail for the dynamics of simply supported rectangular thin plates with stress free edges under influence of a relatively moving mass. By expansion of the solution as a series of mode functions, the governing equations of motion are reduced to an ordinary differential equation for time development vibration amplitude, which is Duffing's oscillator with time varying coefficients. The technique is flexible and could easily be extended for analysis of plates, with various boundary conditions, carrying a moving mass. Schauder's second fixed point theorem, as discussed above, is used which includes the response of a plate under moving force systems. Finally, some numerical examples are solved and the validity of the present analysis is clearly shown, then a parametric study is developed.

3.1 Formulation of the problem

A rectangular plate with a moving mass and simply supported boundary conditions is considered. A mass with considerable inertia is moving on an arbitrary trajectory $x = x_0(t)$, $y = y_0(t)$ on the plate surface, as shown in Fig. 1. The modulus of rigidity, *D*, and the mass per unit volume of the plate, ρ *(x,y)*, are assumed to be constant with no damping present in the system. It is further assumed that the mass moving on the plate is a constant quantity. The governing equation of the finite amplitude motion of a rectangular plate shown in Fig. 1 under mass *M* may be written as [24]:

$$\nabla^4 w + \frac{\rho h}{D}\frac{\partial^2 w}{\partial t^2} = \frac{p(x,y,t)}{D} + \frac{h}{D}\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 \Phi}{\partial y^2} - 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 \Phi}{\partial y \partial x} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 \Phi}{\partial x^2}\right] \tag{30}$$

$$\nabla^4 \Phi = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{31}$$

in which:

$$p(x,y,t) = q(x,y,t) + M\left(g - \frac{\partial^2 w}{\partial t^2}\right)\delta(x - x_0(t))\delta(y - y_0(t)) \tag{32}$$

In derivation of Eqs (30) and (31), both extensional and rotary inertia effects have been neglected. For simply supported rectangular plate of sides *a* and *b*, the boundary conditions on w are,

$$\begin{aligned} w &= 0, \quad v\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} = 0 \qquad (x = 0, a) \\ w &= 0, \quad \frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2} = 0 \qquad (y = 0, b) \end{aligned} \tag{33}$$

The boundary conditions for the Airy stress function for the case of a plate with stress free edges become,

$$\begin{aligned} \frac{\partial^2 \Phi}{\partial y^2} &= \frac{\partial^2 \Phi}{\partial y \partial x} = 0 \qquad (x = 0, a) \\ \frac{\partial^2 \Phi}{\partial x^2} &= \frac{\partial^2 \Phi}{\partial y \partial x} = 0 \qquad (y = 0, b) \end{aligned} \tag{34}$$

For simply supported rectangular plate of sides *a* and *b*, an approximate single mode assumption is employed in order to derive the decoupled equations for the dynamics of the time dependent amplitudes.

$$w(x,y,t) = h.A(t).\sin\left(\frac{m\pi x}{a}\right).\sin\left(\frac{n\pi y}{b}\right) \tag{35}$$

where *A (t)* is a dimensionless function of time, and m and n are integers. The function (35) obviously satisfies the boundary conditions (33). Inserting Eq. (35) in (31) yields

$$\nabla^4 \Phi = \frac{E}{2}\left(\frac{nm\pi^2 h}{ab}\right)^2 \times (A(t))^2\left[\cos\left(\frac{2m\pi x}{a}\right) + \cos\left(\frac{2n\pi y}{b}\right)\right] \tag{36}$$

The Ritz–Galerkin method is used to obtain the solution of Eq. (36) and Eq. (30). The following solution for the stress function is assumed:

$$\Phi = c.h^2.A^2(t).\left(1 - \cos\left(\frac{2m\pi x}{a}\right)\right)\left(1 - \cos\left(\frac{2n\pi y}{b}\right)\right) \tag{37}$$

The assumed solution (Eq. 37) clearly satisfies the stress free edges boundary conditions (Eq. 34). More general approaches to find the Airy functions could be found in the literature. The expressions for the stress function Φ and the values for w (Eq. 35) satisfy the boundary conditions and Eq. 31. One cannot, however, expect that they will also exactly satisfy Eq. (36). Employing Eq. (37) with Eq. (36) results:

$$c = -\frac{E(mn/ab)^2}{8\left[(m/a)^2 + (n/b)^2\right]^2} \tag{38}$$

Employing Eqs (35) and (37) into Eq. (30), multiplying the resulting Equation by the corresponding selected mode shape of the lateral displacement and integrating over the surface of the plate results,

$$\ddot{A} + \frac{\pi^4 D}{\rho\alpha(t)h}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2 A = \frac{4R_{mn}(t)}{\rho\alpha(t)h^2 ab} - \frac{Eh^2}{2\rho\alpha(t)}\left(\frac{mn\pi}{ab}\right)^4 \times \frac{A^3}{\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2} \tag{39}$$

where:

$$R_{mn}(t) = \int_0^a \left\{ \int_a^b [q(x,y,t) + Mg.\delta(x - x_0(t))\delta(y - y_0(t)) \times \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\right] dy \} dx \quad (40)$$

And

$$\alpha(t) = 1 + \frac{4M}{ab\rho h}\sin^2\left(\frac{m\pi x_0(t)}{a}\right)\sin^2\left(\frac{n\pi y_0(t)}{b}\right) \quad (41)$$

Defining of the following time varying parameters:

$$P(t) = \frac{\pi^4 D}{\rho\alpha(t)h}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2 \quad (42a)$$

$$Q(t) = \frac{Eh^2}{2\pi\alpha(t)}\left(\frac{mn\pi}{ab}\right)^4 \Big/ \left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^2 \quad (42b)$$

$$F(t) = \frac{4R_{mn}(t)}{\rho\alpha(t)h^2 ab} \quad (42c)$$

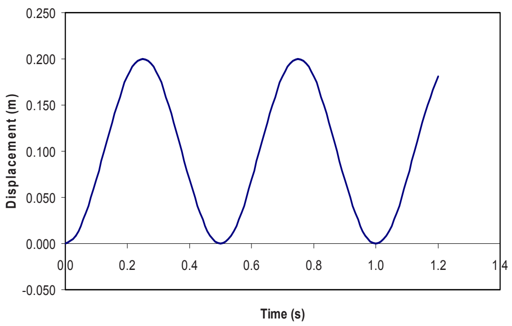

Fig. 2. Non-dimensional displacement of center of nonlinear rectangular plate for the first example as a function of time.

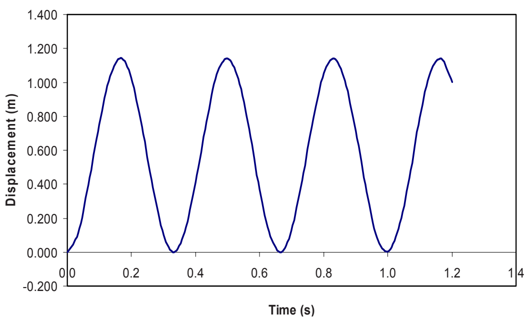

Fig. 3. Non-dimensional displacement of center of nonlinear rectangular plate for the second example as a function of time.

Eq. (39) can be rewritten as,

$$\ddot{A} + P(t)A + Q(t)A^3 = F(t) \quad (43)$$

Eq. (43) belongs to the class of "Duffing Equations with time dependent coefficients. The coefficients are periodic if the loading is periodic. The results of two problems of these kinds are listed in [25].

3.2 Case studies

With the view of illustrating the theoretical results obtained in this research, we now present two numerical studies.

Case 1: The plate chosen for the purpose of numerical evaluation is a simply supported square plate. The physical data for such example is given in Table-1. It is excited by a mass rotating with a constant angular velocity equal to *12.567 rad/s* on a circle of radius *1.5m* centered at the center of the plate. It is desired to determine the transient response of the plate for the first *0.5* second time interval that is time necessary to complete one orbiting cycle. It is assumed that the plate is originally at rest. The numerical values for displacement of the center point of the nonlinear plate are shown in Table-2 as a function of time and are illustrated diagrammatically in Fig. 2.

Since $T^2P^* = 10.96$, i.e., $o < T^2P^* < 16$, there is a guarantee for the existence of a periodic solution according to the above discussion, as it is demonstrated in Fig. 2.

Case 2: As the second example, steel plate with the following properties is considered. The dimensions and other properties are shown in Table-1. The mass of the moving load is moving on an inscribed ellipse, in the plate with constant angular speed of *18.791 rad/s*. Only first vibration mode of the plate is considered (*m=1*, *n=1*). The example is solved via numerical integration of Eq. (39) and the achieved result is illustrated in Fig. 3. In this case T and P^* can be calculated to be *0.332s* and *203s*$^{-2}$, respectively, and T^2P^* is *22.5*. So the above discussion does not guarantee the existence of a periodic solution, though from Fig. 3 it seems to have a periodic solution. Table-2 as well as Fig. 3 shows the results.

3.3 Parametric study

In this section a simply supported rectangular square plate with the same properties like case 1, is considered. A parametric study has been carried out

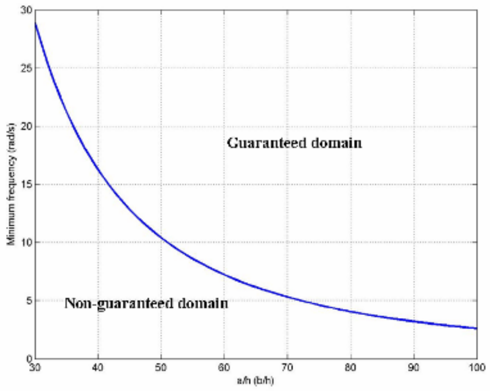

Fig. 4. Minimum frequency of moving load vs. (a/h=b/h), in which the existence of periodic solution is guaranteed through this study.

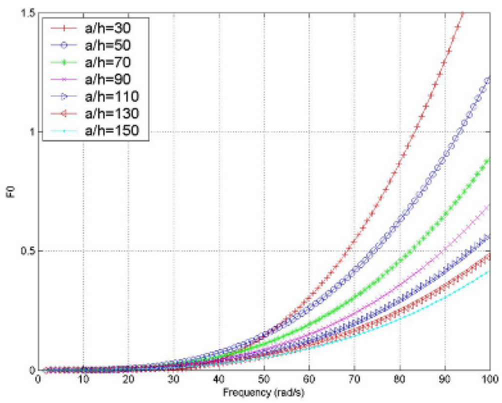

Fig. 5. Dependence of maximum allowable vibrating force (N) on frequency of moving load and aspect ratio of the rectangular plate (a/h=b/h).

and the results are given in Figs 4-5.

In Fig. 4 the range of the problem parameters for which the existence of periodic solution is guaranteed based on the application of the second Shauder's fixed-point theorem is presented and the effects of geometrical properties (width to thickness ratio) of the plate to the minimum frequency of moving load are shown. It can be inferred that the thinner the plate, the smaller minimum frequency and the larger domain will exist.

Also, the dependence of the moving mass frequency to the maximum applied vibrating force (F_0) for the various geometrical properties (width to thickness ratio) of the plate is shown in Fig. 5. It is found that increasing the frequency of moving mass would increase the domain for applied frequency in which the periodic response is guaranteed for the various

Table 1. geometric and physical properties of plate.

	case 1	**case 2**
$\rho\left(kg/m^2\right)$	*2,133*	*2,770*
$D\left(kg/m\right)$	*15,000*	*15,000*
a/h	*50*	*120*
b/h	*50*	*60*
$h\left(m\right)$	*0.1*	*0.05*
$M\left(kg\right)$	*2000*	*2000*

Table 2. Numerical results for displacement of center point of plate (W_0).

case 1		case 2	
Time (s)	W_0 (m)	Time (s)	W_0 (m)
0.000	*0.000*	*0.000*	*0.000*
0.067	*0.033*	*0.033*	*0.11*
0.133	*0.11*	*0.067*	*0.398*
0.2	*0.181*	*0.1*	*0.752*
0.267	*0.198*	*0.133*	*1.038*
0.333	*0.15*	*0.167*	*1.142*
0.467	*0.009*	*0.233*	*0.738*
0.533	*0.009*	*0.267*	*0.384*
0.6	*0.069*	*0.3*	*0.102*
0.667	*0.15*	*0.667*	*0.3*
0.733	*0.198*	*0.733*	*0.396*
0.8	*0.181*	*0.8*	*0.362*
0.867	*0.11*	*0.867*	*0.22*
0.933	*0.033*	*0.933*	*0.066*
1.000	*0.000*	*1.000*	*0.000*
1.067	*0.033*	*1.067*	*0.066*
1.133	*0.11*	*1.133*	*0.22*
1.2	*0.181*	*1.2*	*0.362*

geometrical properties of the plate. Also, it is illustrated that increasing the geometrical ratio of the plate (*a/h*) would result in decreasing the maximum allowable vibrating force (F_0). In the other words, the thinner the plate is, the smaller limit for vibrating force will exist.

4. Conclusion

This paper undertakes Schauder's second fixed point theorem, which includes the response of structures under vibratory force systems. The periodicity of the overall nonlinear relations as Duffing's oscillator with time varying coefficients was investigated and the sufficient conditions for the existence of periodic oscillatory behavior were obtained. Proofing the theorem of the existence of periodic solution, we worked in the Banach space and used Schauder's second fixed point theorem, as well as Arzela-Ascoli's relative compactness theorem. While existence of the response is guaranteed in the resulting domain, the method does not say anything about the response outside this sufficient condition. This is illustrated by an example. Based on classical plate theory a rectangular plate on which a moving mass is traveling with specific frequency is considered. The plate is supposed to be simply supported and large deflection (middle plate stretching) has been taken into account. By expansion of the solution as a series of mode functions, the governing partial differential equation was converted into a nonlinear ordinary differential equation with time varying coefficients. Similar equations can arise in the analysis of other mechanical systems. Regular response of dynamical systems has many applications in control of engineering systems. These conditions guarantee the existence of a periodic response with the same frequency of the base excitation, for which the plate is initially at rest. The range of the parameters of the problem for which the existence of periodic solution is guaranteed is presented and the effects of the geometrical properties of the plate as well as the maximum allowable vibrating force to the minimum frequency of moving load were studied in detail.

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Nomenclature

A (t)	:	Dimensionless function of time
a	:	Plate length
b	:	Plate width
c	:	Ritz unknown constant
D	:	Bending stiffness
E	:	Young's modulus of elasticity
f^*	:	Force in Hammerstein Equation
g	:	Gravity
g (t, ζ)	:	Green's function
h	:	The thickness of the plate
K (s, ζ)	:	To be defined in text and also Hammerstein Equation
L	:	Functional Operator
M	:	Mass
$\bar{M}$	:	Any positive number
p (x,y,t)	:	Lateral distributed force on the plate
q (x,y,t)	:	General dynamic loading without inertial effects
R	:	Relatively compact set
S	:	Banach space
s	:	Rescaled variable
T	:	Period of response
t	:	Time
w (x,y,t)	:	Lateral deflection of the plate
x-y	:	Cartesian coordinate
x	:	Reference variable along horizontal direction
y	:	Reference variable along vertical direction
β	:	Rescaling Factor
δ	:	Dirac Delta function
Φ	:	Airy stress function
ν	:	Poisson's ratio
ρ (x,y)	:	Density of the plate
ω	:	Load Frequency
ζ	:	Generalized coordinate
'	:	Partial derivative with respect to the rescaled variable (s)
.	:	Partial derivative with respect to time (t)

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